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Optical solitons in erbium-doped fibres with higher-order effects and pumping

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Abstract. We consider the coupled system of the higher-order nonlinear Schrödinger equation and Maxwell–Bloch equations with pumping, which governs the nonlinear wave propagation in erbium-doped optical waveguides in the presence of important higher-order effects. We derive the Lax pair with a variable spectral parameter and the exact soliton solution is generated from the Bäcklund transformation.

The dynamics of a nonlinear short-optical-pulse envelope in a fibre is described by

$$\mathrm{i}q_z + \frac{k''}{2}q_{tt} + \beta|q|^2q + \frac{\mathrm{i}k'''}{6}q_{ttt} + \mathrm{i}\gamma(|q|^2q)_t + \mathrm{i}\gamma_s(|q|^2)_t q = 0 \tag{1}$$

where q represents the complex envelope amplitude, t and z are the time and distance along the direction of propagation, k'' is the second derivative of the axial wavenumber k with respect to the angular frequency ω_0 and describes group velocity dispersion (GVD), $k''' = \partial^3 k/\partial\omega^3$ at ω_0 describes higher-order dispersion, $\beta = n_2\omega_0/cA_{\mathrm{eff}}$ is the self-phase modulation (SPM) parameter, where n_2 is the Kerr coefficient and c is the speed of light, A_{eff} is the effective core area of the fibre, $\gamma = 2\beta/\omega_0$ describes Kerr dispersion (also called self-steepening) and γ_s represents the delayed nonlinear process. The imaginary part of γ_s describes stimulated Raman scattering.

Equation (1) becomes the nonlinear Schrödinger (NLS) equation when terms proportional to k''', γ and γ_s are negligible [1–3]. However, in some regions, the role of k''' becomes important. In particular, to describe the effects of pulse broadening in the frequency region where k'' is close to zero, one needs to take k''' to be non-negligible [4, 5]. The last two terms proportional to γ and γ_s become important for short-pulse propagation over long distances [6, 7]. Physically, GVD and higher-order dispersion are linear effects, which spread the pulse temporally. SPM, Kerr dispersion and delayed nonlinear process are nonlinear effects, which spread the pulse in the frequency domain. Unlike GVD and SPM, higher-order dispersion, Kerr dispersion and the delayed nonlinear process spread the pulse asymmetrically. In the absence of higher-order effects, GVD and SPM balance each other in the anomalous dispersion regime to form optical bright solitons. Equation (1) as such, with all the effects, is called the higher-order nonlinear Schrödinger (HNLS) equation. In [8, 9], the HNLS equation is considered and soliton solutions have been derived for a particular condition.

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In 1967, McCall and Hahn [10] explained a special type of lossless pulse propagation in two-level resonant media. They showed that if the energy difference between the two levels of the medium coincides with the optical wavelength, then coherent absorption takes place. The medium becomes optically transparent to that particular wavelength, which is called self-induced transparency (SIT). Maxwell–Bloch (MB) equations explain the process of SIT. In [11] MB equations with pumping and damping have been considered and the explicit form of the Lax pair has been presented with a variable spectral parameter. Burtsev and Gabitov [11] have clearly explained the need for optical pumping during the propagation of optical pulses in resonant atoms.

If the fibres are doped with erbium atoms, then SIT can also be induced in optical fibres. This type of soliton pulse propagation was shown for the first time by Maimistov and Manykin [12] in 1983 and many other results were also reported on the NLS-MB fibre system [13–20]. Nakazawa *et al* [21, 22] experimentally observed the coexistence of NLS solitons and SIT solitons in erbium-doped resonant fibres. In [23–25], the possibility of coexistence of an NLS soliton and an SIT soliton with some higher-order terms are also shown. We have considered the HNLS-MB equations already [23, 25] and shown that they allow soliton-type pulse propagation for a particular choice of parametric condition.

Here, in this paper, we consider the HNLS-MB equations with pumping and show that, for a reduced dynamical equation, the erbium-doped fibre system allows soliton-type pulse propagation with pumping. The explicit form of the Lax pair for HNLS-MB equations with pumping is presented. To derive the initial soliton pulse shape and speed, the exact soliton solution is derived from the Bäcklund transformation.

HNLS-MB equations with pumping take the form

$$\begin{aligned} &\mathrm{i}q_z + \frac{k''}{2}q_{tt} + \beta|q|^2q + \frac{\mathrm{i}k'''}{6}q_{ttt} + \mathrm{i}\gamma(|q|^2q)_t + \mathrm{i}\gamma_s(|q|^2)_t q = 2\mathrm{i}\langle p\rangle \\ &p_t = 2\mathrm{i}\omega p + 2q\eta \\ &\eta_t = -(qp^* + q^*p) - c \end{aligned} \tag{2}$$

where p and η are given by $\nu_1\nu_2^*$ and $|\nu_1|^2 - |\nu_2|^2$ respectively (ν_1 and ν_2 are wavefunctions of two energy levels of erbium atoms). The bracketed term $\langle\cdots\rangle$ denotes averaging over the entire frequency range,

$$\begin{aligned} &\langle p(z,t;\omega)\rangle = \int_{-\infty}^{\infty} p(z,t;\omega)g(\omega)\,\mathrm{d}\omega \\ &\int_{-\infty}^{\infty} g(\omega)\,\mathrm{d}\omega = 1 \end{aligned} \tag{3}$$

where $g(\omega)$ is the uncertainty in the energy levels of the erbium atoms.

Kodama [26] has shown that with suitable transformation and omitting the higher-order terms and with the condition $k'' = \beta$, HNLS equation (1) reduces to the Hirota equation [27]. In a similar way equation system (2) can be reduced to a coupled system of the Hirota equation and MB equations with pumping of the following form:

$$\begin{aligned} &q_z = \mathrm{i}\beta(\tfrac{1}{2}q_{tt} + |q|^2q) + \epsilon(q_{ttt} + 6|q|^2q_t) + 2\langle p\rangle \\ &p_t = 2\mathrm{i}\omega p + 2q\eta \\ &\eta_t = -(qp^* + q^*p) - c. \end{aligned} \tag{4}$$

Here we have considered only a special choice of parametric values for the coefficients of the higher-order terms for which the equation system is completely integrable.

The linear eigenvalue problem associated with equation (4) is constructed as

$$\frac{\partial \Psi}{\partial t} = U\Psi \qquad \Psi = (\psi_1 \quad \psi_2)^T \tag{5}$$

where

$$U = \begin{pmatrix} -\mathrm{i}\lambda & q \\ -q^* & \mathrm{i}\lambda \end{pmatrix}. \tag{6}$$

λ is the variable spectral parameter given by

$$\lambda_z = \left\langle \frac{c}{\lambda + \omega} \right\rangle \qquad \lambda_t = 0. \tag{7}$$

The space evolution of eigenfunction Ψ is given by

$$\frac{\partial \Psi}{\partial z} = V\Psi \tag{8}$$

$$V = [-4\mathrm{i}\epsilon\lambda^3 + \mathrm{i}\beta\lambda^2 + \epsilon(qq_t^* - q_t q^*)] \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + (-4\epsilon\lambda^2 + \beta\lambda + 2\epsilon|q|^2) \begin{pmatrix} 0 & q \\ -q^* & 0 \end{pmatrix} + \left[-2\mathrm{i}\epsilon\lambda + \mathrm{i}\left(\frac{\beta}{2}\right)\right] \begin{pmatrix} |q|^2 & q_t \\ q_t^* & -|q|^2 \end{pmatrix} + \epsilon \begin{pmatrix} 0 & q_{tt} \\ -q_{tt}^* & 0 \end{pmatrix} + \mathrm{i} \begin{pmatrix} \langle \frac{\eta}{\lambda+\omega} \rangle & \langle \frac{-p}{\lambda+\omega} \rangle \\ \langle \frac{-p^*}{\lambda+\omega} \rangle & \langle \frac{-\eta}{\lambda+\omega} \rangle \end{pmatrix}. \tag{9}$$

The compatibility condition $U_z - V_t + [U, V] = 0$ gives equation system (4).

Thus from the existence of the linear eigenvalue problem it is clear that equation system (4) is completely integrable.

In order to construct the Bäcklund transformation of equation (4), let us write down equation (5) in terms of the Riccati equation. For this purpose, we introduce a new variable (or pseudopotentional)

$$\Gamma = \frac{\psi_1}{\psi_2}. \tag{10}$$

Equation (10) yields

$$\Gamma_t = -2\mathrm{i}\lambda\Gamma + q + q^*\Gamma^2. \tag{11}$$

Now transformations of variables $\Gamma \rightarrow \Gamma'$, $\lambda \rightarrow \lambda'$ and $q \rightarrow q'$ which keep the form of equation (11) invariant are sought. The simplest transformation can be tried by setting $\Gamma' = \Gamma$, $\lambda' = \lambda^*$ and looking for q' in the form

$$q - q' = \frac{2\mathrm{i}(\lambda - \lambda^*)\Gamma}{1 + |\Gamma|^2}. \tag{12}$$

Equation (12) defines the Bäcklund transformation for equation (4). Here the primed quantities refer to N-soliton solutions and the unprimed quantities refer to $(N - 1)$-soliton solutions. To construct the soliton solution of equation (4), we start with the zero-soliton solutions $q = p = 0$ and $\eta = \pm 1$ (pure states). By substituting the above conditions in the spatial and temporal eigenproblems, the explicit forms of $q(1)$, $p(1)$, $\eta(1)$ and $\Gamma(0)$ are obtained. This procedure can obviously be continued; it furnishes in a recursive manner all the higher-order soliton solutions and the associated wavefunction can also be generated. For instance, the solution of equation (12) is found to be (with $\lambda = \alpha + \mathrm{i}\zeta$)

$$\Gamma(0) = \exp[\psi(z, t) + \mathrm{i}\theta(z, t)] \tag{13}$$

where $\psi(z,t)$ and $\theta(z,t)$ are given by

$$\psi(z,t) = 2\zeta t - 2\int\left[4\epsilon\zeta(3\alpha^2 - \zeta^2) + 2\beta\alpha\zeta - \int_{-\infty}^{\infty}\frac{\zeta}{\zeta^2 + (\alpha-\omega)^2}g(\omega)\,\mathrm{d}\omega\right]\mathrm{d}z \tag{14}$$

$$\theta(z,t) = 2\alpha t + 2\int\left[4\epsilon\alpha(\alpha^2 - 3\zeta^2) + \beta(\alpha^2 - \zeta^2) - \int_{-\infty}^{\infty}\frac{(\alpha-\omega)}{\zeta^2 + (\alpha-\omega)^2}g(\omega)\,\mathrm{d}\omega\right]\mathrm{d}z. \tag{15}$$

So one can generate a new set of one soliton solution for equation (4) from (12), which is obtained from the trivial one,

$$q(z,t) = 2\zeta\,\mathrm{sech}(\psi)\exp(\mathrm{i}\theta) \tag{16}$$

$$p(z,t;\omega) = \frac{2\zeta[\zeta\sinh(\psi) + \mathrm{i}(\alpha-\omega)\cosh(\psi)]\exp(\mathrm{i}\theta)}{\zeta^2\sinh(\psi) + (\alpha-\omega)^2\cosh^2(\psi) + \zeta^2/4} \tag{17}$$

$$\eta(z,t;\omega) = \frac{\zeta^2\sinh^2(\psi) + (\alpha-\omega)^2\cosh^2(\psi) - \zeta^2/4}{\zeta^2\sinh^2(\psi) + (\alpha-\omega)^2\cosh^2(\psi) + \zeta^2/4}. \tag{18}$$

Here α and ζ are velocity parameters related to the soliton pulse.

Thus, from the explicit form of the soliton solutions (16)–(18), one can calculate the initial pulse shape, soliton velocity, pulse intensity etc. In equations (16)–(18), if we take the limit $\epsilon \to 0$, then we obtain the soliton solution for the coexistence of NLS solitons and SIT solitons with pumping. For the condition $\lambda_z = 0$, the soliton solution (16)–(18) goes to the limit of solitons in erbium-doped fibres with higher-order effects without pumping [24].

To conclude, we have considered the HNLS-MB equations with pumping which describe the wave propagation of a nonlinear optical field in an erbium-doped fibre medium with important higher-order effects. The linear eigenvalue problem associated with the equation system is derived with a variable spectral parameter. The exact form of the soliton solution is also derived using the Bäcklund transformation.

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